BC Calc Vector Practice 2 **Name:** key 2025 Seat:

1. (NC) A particle moves in the xy-plane so that at any time t, the position of the particle is given by $\mathbf{r}(t) = \langle t^3 + 4t^2, t^4 - t^3 \rangle$.

   (a) Find the velocity vector when $t = 1$.

$\vec{v}(t) = \langle 3t^2 + 8t, 4t^3 - 3t^2 \rangle$

$\vec{v}(1) = \langle 11, -1 \rangle = 11\vec{i} - \vec{j}$

   (b) Find the acceleration vector when $t = 2$.

$\vec{a}(t) = \langle 6t + 8, 12t^2 - 6t \rangle$

$\vec{a}(2) = \langle 12 + 8, 48 - 12 \rangle$

$\vec{a}(2) = \langle 20, 36 \rangle = 20\vec{i} + 36\vec{j}$

2. (NC) A particle moves in the xy-plane so that at any time $t \geq 0$, the position of the particle is given by $\mathbf{r}(t) = \langle t^2 + 3t, t^3 - 3t^2 \rangle$.

(a) Find the velocity vector when $t = 1$.

$\vec{v}(t) = \langle 2t + 3, 3t^2 - 6t \rangle$

$\vec{v}(1) = \langle 2 + 3, 3 - 6 \rangle$

$\vec{v}(1) = \langle 5, -3 \rangle$

$\vec{v}(1) = 5\vec{i} - 3\vec{j}$

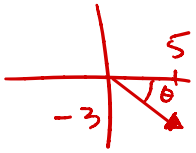
(b) Find magnitude of the velocity vector when $t = 1$

$\| \vec{v}(1) \| = \sqrt{5^2 + 9^2} = \sqrt{34}$

(c) What is the meaning of the magnitude of the velocity vector when $t = 1$

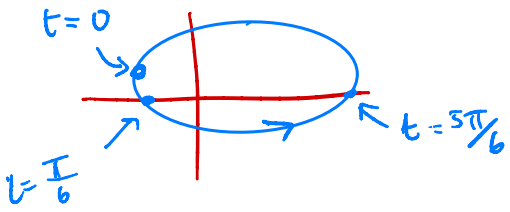
At time $t = 1$ the speed of the particle is $\sqrt{34}$

(By the way the direction is



$\theta = \arctan\left(\frac{-3}{5}\right)$)

3. (NC) A particle moves in the xy-plane at time t, $0 \le t \le 2\pi$, the position of the particle is given by $\mathbf{r}(t) = \langle \sqrt{3} - 4\cos(t), 1 - 2\sin(t) \rangle$. The path intersects the x-axis twice. Write an equation that represents the distance traveled by the particle between the two x-intercepts. Evaluate with your Calculator.



$$y = 1 - 2\sin t = 0$$

$$\sin t = \frac{1}{2}$$

$$t = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\int_{\pi/6}^{5\pi/6} \| \vec{v}(t) \| \, dt$$

$$\int_{\pi/6}^{5\pi/6} \sqrt{(4\sin t)^2 + (-2\cos t)^2} \, dt$$

$$\approx 7.347$$

Answers

1. (a) $< 11, 1 >$, (b) $< 20, 36 >$
2. (a)$< 5, -3 >$ (b) $\sqrt{34}$ (c) at time t=1, the particle is traveling at the speed of $\sqrt{34}$ in the south east direction (the angle $\arctan(-\frac{3}{5})$)
3. intersects when t = $\frac{\pi}{6}$ and $\frac{5}{6}$ so $\int_{\pi/6}^{5\pi/6} \sqrt{(4\sin t)^2 + (-2\cos t)^2}\, dt \approx 7.347$